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GENERALIZED UNICORNS PROBLEM WITH A SPECIAL (α, β)-METRIC

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Abstract: In this paper, we study the generalized unicorns problem on regular (α, β)-metrics in the form of $F = \alpha\phi(s)$, $s = \beta/\alpha$, where α is a Riemannian metric and β is a 1-form on the manifold. We prove that, if $\phi = \phi(s)$ is a special polynomial in s, then F is a weak Landsberg metric if and only if F is a Berwald metric. Further, we prove that if $\phi = \phi(s)$ is a polynomial in s and F is not a Randers metric, then F is of relatively isotropic mean Landsberg curvature if and only if it is a Berwald metric.

Keywords and Phrases: Finsler space, (α, β)-metric, Berwald metric, weak Landsberg metric, generalized unicorns problem.

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1. Introduction

The unicorns problem is partially solved for an important class of Finsler (α, β)-metrics in the form of $F = \alpha\phi(s)$, $s = \beta/\alpha$, where α is a Riemannian metric and β is a 1-form on the manifold M. A Finsler metric F is called Landsberg metric if the Landsberg curvature $L := L_{ijk}dx^i \otimes dx^j \otimes dx^k$ vanishes [7]. A long existing open problem in Finsler geometry is

Is there any Landsberg metric which is not a Berwald metric ?